Modeling Maternal Mortality Data of Some Selected Health Facilities in Rivers State using Poisson Regression Model: (2013 - 2017)

Imo, Dumtochi Department of Mathematics/Statistics, Rivers State University, Nkpolu-Oroworukwo, Port Harcourt

Abstract

The risks associated with pregnancy have been of the greatest challenges that have drawn the attentions of researchers, statisticians, medical practitioners, families and even the pregnant women themselves. These challenges has called for concern among stakeholders, therefore, this study was targeted at modeling maternal mortality data of some selected health facilities in Rivers state using Poisson Regression model. To achieve this target, the study aim of the study was to model maternal mortality rate in some selected Health in Rivers State, while the specific objectives of the study include to: assess factors that contribute to high rate of maternal mortality rate in Rivers State. Secondly, was to fit an appropriate model to data on maternal mortality rate in Rivers State and to select an appropriate fitted model in estimating and modeling of maternal mortality rate in Rivers State. The data for the study was source for and extracted from the Rivers State Hospital Management Board. The data extracted was analyzed using statistical software called STATA version 14. The results obtained revealed that the Poisson regression, *Poisson Generalized Regression, the Negative Binomial Regression and truncated Regression model cannot actually captured the excess Zeros contain in the data and the issue of over dispersion. So, other form of Poisson Regression models such as Zero-inflated Poisson Regression and Zero-inflated Negative Binomial Regression were also used in the estimation. However, zero-inflated Negative Binomial Regression has the least Akaike Information Criterion (AIC) based on the selections of the overall best fitted model. Hence,* recommendation were made base on the results from the findings.

Keyword: Modeling, Maternal, Mortality Health, Facility

Introduction

1.1 Background to the Study

Maternal mortality has become a very significant determinant of human and social development. It is particularly exposes women's overall status, access to health care, and the responsiveness of government to the health care system needs of her citizens. Therefore, awareness of maternal mortality levels is very important not only for identifying the risks associated with pregnancy and childbearing, but also for what it says about women's health and, indirectly, their economic and social status. Determining the level maternal mortality and the associated risk factors is necessary in order to be able to diagnosed issues and assessing the advancement and effectiveness of existing maternity programs in Rivers state and Nigeria at large.

Also, in life women having pregnant and giving birth to children are two physiological events that comes with joy to the women, the husband, family and the society in general as it is a fulfillment of God's commandment that we should go into the world and multiply. However, sometimes in life, the reverse is case as it is occasionally a source of sorrow. According to

Salifu (2004) for some women in certain parts of the world specifically in developing countries, the reality of giving birth to children is often grim. For those women, giving birth to children is often marred by unforeseen challenges that sometimes lead to loss of lives (mother and child). Some women loss the fetus even before being given birth to or immediately after birth, whiles others loss both their lives and the fetus.

In Nigeria today maternal period is prone to crises due to so many factors such as Socio – economic, religious and biological factors all theses interact as well interface with each other. A culmination of these factors couple with other cultural beliefs and practices such gender biasness, low status of women in the society, and high fertility affect child delivery in Nigeria. According to the WHO (2012), maternal mortality could be referred to as the death of women from any cause related to or attributed to pregnancy or its management (excluding) accidental or incidental causes during pregnancy and child bearing or within 42 days of termination of the pregnancy, irrespective of the duration and size of the pregnancy. According to the current estimation by the world health organization (WHO), the United Nation Children Fund (UNICEF) and the World Bank about size hundred thousand (600,000) women die yearly as a result of pregnancy-related complications and 99% of it occur mostly in less developed countries in the world.

Sequel to the above, this situation makes maternal mortality an indicator in the health sector that shows the largest disparity between developed and developing countries in the world. According to MCA lister and Basket (2006), in sub – Saharram Africa one out of every 13 women that gave birth during child bearing dies of pregnancy-related causes during their life time as compare to one in 4085 women in most globalized countries. Also, WHO, UNICEF and World Bank (2008) estimated the number of women that die as a result of pregnancyrelated causes to be in the ratio of one is to 45. They further to suggested that for every maternal death an approximated number of 30 women suffer one form of injuries, complications, infections and disabilities during pregnancy or child bearing in at least in 15 Million women in a year.

It is therefore very obvious that developing countries in the world continue to have the highest number of cases of women that dies as a result of pregnancy- related issues. Sahfu (2014) attributed the causes of these challenges to lack of access to skilled delivery care that will reduce maternal, prenatal mortality and morbidity. The need for reduction in maternal mortality has actually generated a lot of concern among corporate organizations, Non – government organization (NGOS), international community and researchers alike especially in view of its spontaneous increase with a corresponding devastating effects on the women, their families and society in general. The importance of this condition resulted to most notably the launching of the Millennium development goals (specifically) NDG 5: Improving material health), global strategy for women's and children Health in the year 2010 by the United Nations (UN). Secretary – General which was greeted with high profile Global pronouncement.

Subsequently, the high – level commission on information and accountability to determine the most effective and international institution arrangement for global reporting oversight and accountability on women and their children's health, one women and their children's health. One among the ten recommendations of the commission was specifically to improve measure on now to reduce maternal (and children) death. The recommendation required that by 2015 all countries should take steps to establish a system for the registration of births, deaths and causes of death and have well – functioning health information systems that combine data from facilities, administrative sources and survey (WHO, 2014). This provided an opportunity to ensure risk of material mortality is minimized for all concern.

The Government of Rivers State has not done much in this area except the introduction of free antenatal care for all pregnant women from November, 2014 and the exception Laws that was pass which makes child delivery care free that was also put in place.

The driving force of these polices have been to improve uptake, quality financial and geographical access to child delivery care services. According to the Rivers State Government, these services as reflected in the policy covered assisted deliveries such as caesarean section and management of medical complications; normal deliveries and surgical complications due to deliveries for the repair of Visio-vaginal and recto – Vaginal fistulae. Although, this exemption policy (Laws) does not covered delivery services in private and faith – based health facilities (Hospital own by Churches).

In spite of these initiatives by government, corporate and international communities there are still challenges facing women during pregnancy and child bearing. Actually, from the existing literature in this area must emphasis has not been done on the statistical approach to solving this problem using Poisson regression, Negative Binomial Regression, Zero – inflation Poisson and Binomial regression, and truncated regression models. According to Ayumanda *et al,* (2013), all these models fall under the category of the count models those are suitable for analyzing discrete data rate whereby the mean of the distribution is equal to the variance process. Although, there are avalanched of models in modeling maternal mortality rate but most of these models exhibit some limitations. This study, therefore target at filling the existing gaps in literature related to the study, it is against this background that this study among other things was targeted at developed model for modeling material mortality rate in selected Health facilities in Rivers State.

3.1 Methodology

This chapter shall be discussed under the following sub-headlines; Research Design, model specification, justification for the model specification, sources of data, estimation technique and procedures.

3.2 Model Specification

In line with the purpose of the study, the models adopted in the study are Poisson regression, Generalized Poisson Regression, Negative Binomial Regression, Zero-Inflated Poisson Zero-Inflated Generalized Poisson, Zero-Inflated Negative Binomial and The truncated Poisson Regression Model and they are stated as thus;

3.2.1. Poisson Regression Model

According to Nwanko and Nwaigwe (2016), Poisson regression models are generalized linear models with logarithm as the link function. In statistics, the generalized linear model (GLM) is a form of flexible generalization of ordinary linear regression that allows for response variables that have error distributions models other than a normal distribution. The generalized linear model comprises of linear predictor, given as

 $\eta_i = \beta_0 + \beta_1 X_{i1} + \dots + \beta_k X_{ik}$ (3.1)

The two functions are given statistical as thus:

The first link functions describes how the mean $E(Y_i) = \mu I$, depends on the linear predictor $\Theta(\mu_i) = n_1$ (3.2)

The second link functions describes how the variance, $var(Y_i)$ depends on the mean

 $E(Y/X) = e^{XB}$ (3.12)

Suppose Y_i are independent observation with a responding values X_i of the predicted variables, then β can be estimated using the maximum likelihood estimates according to Nwanko and Nwaigwe, (2016) lacks a closet – form of expression and must be estimated by numerical methods. He further opined that the probability surface for maximum – likelihood Poisson regression is always convex such that Newton-Raphson of other gradients –based methods are the appropriate estimation techniques.

Therefore, suppose Y_i is a random variable and it takes non-negative values such that $i = 1, 2$ n, where n is the number of observations. Since y_i follows a Poisson distribution, therefore the probability mass function (PMF) is as thus:

$$
\rho(Y_i = Y_i) = \frac{\lambda_i^{yi} \ell \times \rho(-\lambda_i)}{Y_i!}, \ y_i = 0, 1, 2
$$
\n(3.13)

With mean and variance as

 $E(y_i) = Var(y_i) = \lambda_i$ (3.14)

Where the conditional mean (predicted mean) of the Poisson distribution as given in equation (3.12) above specified as;

$$
E(Y_X) = e^{xB} = \lambda_i = E(y_i)
$$
\n
$$
(3.15)
$$

Where it is the value of the explanatory variable $\beta = (B_1, B_2,...B_k)$ are unknown K – dimensional vector of regression parameters. The mean of the predicted Poisson distribution is given as $E(Y/x)$ and its corresponding variance of Y_i as var (Y/x) .

Maximum Likelihood Estimation of the Parameter (β) in Poisson Regression Model **above**

According to Nwankwo and Nwaigwe (2016), the parameter β can be estimated using the maximum likelihood estimation method (MLE). Given that

$$
\ell(\beta) = \prod_{c=1}^{n} \frac{\lambda_i^{y_i} e^{-\lambda_i}}{y^j}
$$
\n(3.16)

The log-likelihood function id given as this

$$
\text{In } \ell(\beta) = \sum_{i=1}^{n} \left[-\lambda_i + y_i \text{ In } \lambda_i - \ln y_i! \right] \tag{3.17}
$$

Recall that $\lambda_i = \ell^{x\beta}$ $\mathbf{v}_i = e^{x\beta}$, hence substitute it in the above equation we have.

$$
In \ell(\beta) = \sum_{i=1}^{n} \left[y_i (x\beta) - \ell^{x\beta} - In \ y_i! \right]
$$
 (3.18)

Differentiating equation (3.18) with respect to β and equating the derivative to zero, we have

$$
\frac{\partial \ln \ell(\beta)}{\partial \beta} = \sum_{i=1}^{n} (y_i - \exp(x\beta))x = 0, i = 1, 2 \cdots k
$$
\n(3.19)

This yields K-nonlinear equations and can be solved using the Newton-Raphson method.

Also, it is noted that where.

3.2.2 Generalized Poisson Regression Model

The Generalized Poisson Regression model According to Nwankwo (2016), one of the advantage of using the generalized Poisson regression model is that it can be fitted for condition of over-dispersion i.e. where $var(y_i) > E(y_i)$ as well as under-dispersion, $var(y_i) < E(y_i)$. Famoye (1993) Wang and Famoye (1997) suggested that when y_i is a count response variable and its follows a Generalized Poisson distribution, the probability density function of y_i , $i = 1, 2, \ldots$ n is given as

$$
f(y_i) = \rho(y_i = y_i) = \left[\frac{\mu_i}{1 + \alpha_i \mu_i}\right]^{y_i} \frac{(1 + \alpha y_i)^{y_i}}{y_i!} \exp\left[\frac{V_i(1 + \alpha y_i)}{(1 + \alpha \mu_i)}\right] y_i = 0, 1, \quad (3.20)
$$

where, mean $E(y_i) = \mu_i$ and variances var(y_i) = ($\mu + \alpha, \mu_i$)²

Where μ is called the dispersion parameter the generalized Poisson distribution is a natural extension of the Poisson distribution (Nwakwo and Nwaigwe, 2016). When $\alpha = 0$, equation (3.21) reduces to the Poisson (as in equation 3.13), resulting to var(y_i) = E(y_i). when $\alpha > 0$, it means var(y_i) and the distribution represents count data with over-dispersion if $\alpha > 0$, it means $var(y_i)$ < Eayi), the distribution represents count data with under-dispersion. Supposing it is assumed that the mean of the fitted value is multiplication i.e. $Ecy_i(x) \mu_i = e_i exp(\phi_i \beta)$ Where eⁱ denotes a measure of exposure similarly xi apx1 vector of explanatory variables and B a Px1 vector of regression parameters (Nwonkwo and Nwaigwe, 2016).

Maximum Likelihood Estimated of the Parameters $\stackrel{\wedge}{(\alpha},\beta)$ of the Generalized Poisson **Regression Model.**

According to Wang and Famoye (1997), the long-likelihood functions of the generalized

Poisson Regression model is define as this;
\n
$$
\ell(\beta, \alpha) = \sum_{i=1}^{n} y_i \log \left(\frac{\mu_i}{1 + \alpha \mu_i} \right) + (y_i - 1) \log (1 + \alpha y_i) - \mu_i \left(\frac{1 + \alpha y_i}{1 + \alpha \mu_i} \right) - \log (y_i)
$$
\n(3.21)

Hence, the maximum likelihood estimated (α, β) will be obtained by maximizing $\ell(\beta, \alpha)$ with respect to β and α . The given equations are as follows:

Let
$$
E(y_i / x_i) = \mu_i = \exp(x_i \beta)
$$
, taking the partial derivative for
\n
$$
\frac{\partial \ell(\beta, \alpha)}{\partial \beta} = \sum_{i=1}^n \frac{(y_i - \mu_i)\mu_i x_{ij}}{(1 + \alpha \mu_i)^2} = 0 \quad j = 1, 2, \dots, \rho
$$
\n(3.22)

Similarly, the partial derivative for
$$
\alpha
$$

\n
$$
\frac{\partial \ell(\beta \alpha)}{\gamma \beta} = \sum_{i=1}^{n} -\frac{y_i \mu_i}{1 + \alpha \mu_i} + \frac{y_i (y - 1)}{1 + \alpha y_i} - \frac{(\ell_{ji} - \mu_i) \mu_i}{(1 + \alpha \mu_i)^2} = 0
$$
\n(3.23)

According to Nwankwo and Nwaigwe (2016), the parameter α and β are estimated by the Newton – Raphson method. In like manner, it can also be estimated using method of moment. This will involved equating the Poisson chi-square statistic with (n-p) degree of freedom, as it was suggested in Breslow (1990) and it is stated as thus;

$$
\sum_{i=1}^{n} \frac{(y_i - \mu_i)^2}{\mu_i (1 + \alpha \mu_i)^2} = n - p \tag{3.24}
$$

Where n is the number of values and p is the member of regression parameters.

3.2.3 Negative Binomial Regression (NBR)

Also, another model used in this study was the Negative Binomial Regression (NBR) model. One important application of the negative binomial regression, according to Nwanko *et al.,* (2016) is that it comprises of a mixture of a family of Poisson distributions with GAMA mixing weights. It is seen as a generalization to Poisson distribution. It is a form of regression whereby the Poisson parameter itself is considered a random variable, distributed according to a GAMA distribution under and so, it is otherwise referred to as Poisson – GAMA mixture. As it is an alternative to Poisson regression, the negative binomial regression captured the issues of overdispersion by incorporating a dispersion parameter to accommodate an unobserved heterogeneity in the count data. According to Greenwood and Yule (1920), as the name of the distribution implies i.e. Poisson-GAMA, it is a mixture of two distribution of the same family of functions that has a closed form which leads to the negative binomial distribution. Cook (2009), observed that the name of this distribution came into being by the application of the binomial theorem with a negative exponent. These two regressions were developed to measure over-dispersion that are commonly observed in discrete or count data (Lord et al, 2005). Similarly, the model is specified as thus:

Supposing we have a series of random counts variables that follows a Poisson order such that:

$$
F(Y_{ij}, \lambda_i) = \frac{\ell^{-\lambda_i} \lambda_i^y}{y_{ij}}, \quad y \ge 0, \lambda \ge 0
$$
\n
$$
(3.25)
$$

Where y_i is the observed number of counts for $i = 1, 2, \dots$ n and λ_i is the mean of the Poisson.

According to Cameron and Trivedi (1998), supposing the mean is assumed to have a random intercept term and the said term is a conditional mean function in its multiplication order then we have the following relationship.

$$
\lambda_i = \exp(\beta_0 + \sum_{j=1}^k X_{ij} \ \beta_j + \varepsilon_i)
$$
\n(3.26)

$$
\lambda_i = \ell \sum_{j=1}^k X_{ij} \beta_j \ell(\beta_0 + \varepsilon_i)
$$
\n(3.27)

$$
\lambda_i = \ell \left(\beta_0 + \sum_{j=1}^k X_{ij} \beta_j \right) \ell^{(\varepsilon_i)} \tag{3.28}
$$

$\lambda_i = \mu_i V_i$

Where $\ell^{(a)}$ - gamma $(\alpha^{-1}, \alpha^{-1})$ $\ell^{(\beta_0 + \varepsilon_i)}$ is define as a random intercept. $\mu_i = \exp(\beta_0 + \sum_{j=1}^k X_{ij} \beta_j)$ is the log-link between the Poisson mean and the covariates of independent variables X_s^o and β 's are the w-efficient of the regression. The marginal distribution of y_i can be obtained by integrating the error term, V_i Therefore, $f(y_i,\mu_i)$ = $\int_0^\infty g(y_i, \mu_i, V_i) h(V_i) dv_i$ $\int_{0}^{\infty} g(y_i, \mu_i, V_i) h(V_i) dv_i$

$$
f(y_i, \mu_i) = Ev[g(y_i, \mu_i, V_e)] \tag{3.29}
$$

where h(V_i) is a mixing distribution. In the case of the Poisson – GAMMA mixture, $g(y_{ii}, \mu_{ii})$ V_i) is the Poisson distribution and $h(V_i)$ is the GAMMA distribution component of the model. The closed form that leads to negative binomial distribution are given as thus: Supposing the variance V_i have two parameters of Gamma distribution.

$$
h(v_{ij} \alpha, \delta) = \frac{\delta^{\alpha}}{\Gamma(\alpha)} V_i^{\alpha - 1} \ell^{-\nu i \delta} \quad \alpha > 0, \delta > 0, V_i > 0
$$
\n(3.30)

Where $E(V_i) = \frac{\alpha}{\delta}$ $\frac{\alpha}{\delta}$ and Var(v_i) = $\frac{\alpha}{\delta^2}$ $\frac{\alpha}{\delta^2}$, Let $\alpha = \delta$, we have , E(v_i) = 1 and var(v_i) = $\frac{1}{\delta}$ 1

Cameron and Trivedi (1998) observed that the transformation of the gamma distribution as a function of the Poisson mean gives a probability density function (pdf) as;

$$
h(\lambda_i, \alpha, \mu) = \frac{(\alpha/\mu_i)\alpha}{\Gamma(\alpha)} \lambda_i^{\alpha-1} \ell \frac{-\lambda_i}{\mu_i} \delta
$$
\n(3.31)

Combating equations (3.30) and (3.31) into equation (3.29) gives the marginal distribution of yi:

$$
f(y_i, \alpha_i, \mu_i) = \int_0^\infty \frac{\exp(-\lambda)\lambda_i^{y_i}}{y_i!} \frac{\left(\frac{\alpha}{\mu_i}\right)^\alpha}{\Gamma(\alpha)} \lambda_i^{a-1} \ell \frac{-\lambda_i}{\mu_i} \delta d\lambda_i
$$
 (3.32)

Using the properties of the gamma function and let $\delta = \alpha$, hence equation (3.31) is reduce to the define equation given as

$$
f(y_i, \alpha_i, \mu_i) = \frac{\left(\alpha \mu_i\right)^{\alpha}}{\Gamma(\alpha)\Gamma(y_i + 1)} \int_0^{\alpha} \exp\left(-\lambda_i \left(1 + \frac{\alpha \mu_i}{\mu_i}\right)\right) \lambda^{yi + \alpha - 1} d\lambda \tag{3.33}
$$

α

$$
f(y_i, \alpha_i, \mu_i) = \frac{\left(\alpha/ \mu_i\right)^{\alpha} \left(1 + \alpha/ \mu_i\right)^{-(\alpha + y_i)} \Gamma(\alpha + y_i)}{\Gamma(\alpha)\Gamma(y_i + 1)}
$$
(3.34)

$$
f(y_i, \alpha_i, \mu_i) = \frac{\Gamma(y_i + \alpha)}{\Gamma(\alpha)\Gamma(y_i \times 1)} \left[\frac{\alpha}{\mu_i \times \alpha}\right]^{\alpha} \left[\frac{\mu_i}{\mu_i \times \alpha}\right]^{y_i}
$$
(3.35)

Therefore, equation (3.35) above become the pdf of the Negative Binomial Regression. Also lord and park (2014) suggested that the mean and variant are as follows; $E(y_i, \alpha, \mu_i) = \mu_I$ (3.36)

Var(yi,
$$
\alpha
$$
, μ_i) = $\mu_i + \frac{\mu^2}{\alpha}$ (3.37)

After the above links between models have been established, the next step involved the definition of the log-likelihood function and this is demonstrated in lord and park (2013) as thus;

$$
\ln\left[\frac{\Gamma(y_i+\alpha)}{\Gamma(\alpha)}\right] = \sum_{j=0}^{y-1} In(j+\alpha)
$$
\n(3.38)

Substituting equation (3.28) and (3.26) , the log-likelihood function can be computed using the logarithmic function givens as;

$$
In(L(\alpha, \beta) = \sum_{i=1}^{n} \left\{ \left[\sum_{j=0}^{y-1} In(j+\alpha) \right] = In \ y_i^1 - (yi+\alpha)In(1+\alpha^{-1}\mu) + y_i \ In \ \alpha-1+y_iIn \ \mu_i \right\} \right\}
$$
(3.39)
Hence, the log-likelihood function becomes

$$
n(L(\alpha, \beta) = \sum_{i=1}^{n} \left\{ y_i In(\frac{\alpha\mu_i}{1-\alpha}) - \alpha^{-1}In(1+\alpha\mu_i) + In \ \Gamma\left(y_i + \alpha^{-1}\right)_i \ \right\} - n\Gamma\left(y+1\right) - In \ F\left(\alpha^{-1}\right) \right\}
$$
(3.40)

Hence, the log-likelihood function becomes
\n
$$
n(L(\alpha, \beta)) = \sum_{i=1}^{n} \left\{ y_i \ Im\left(\frac{\alpha \mu_i}{1 + \alpha \mu_i}\right) - \alpha^{-1} \ Im(1 + \alpha \ \mu_i) + \ Im\ \Gamma\left(y_i + \alpha^{-1}\right)_{i} \ \right\} - n\Gamma\left(y + 1\right) - \ Im\left(\alpha^{-1}\right) \right\}
$$
\n(3.40)

(i) Zero-Inflated Regression Model

The zero-inflated (Zero inflated Poisson, Zero-inflated Generalized Poisson Regression, Zero inflated Negative Binomial Regression) according to Hu, Pavlicova and Nunes (2011) have been developed to deal mostly with the excess zeros in the observed outcome of data. Although, they are quite different in application, interpretations as well as usage in analyzing of count data unlike other model. According to Yusuf, Afocabi and Agbaje (2018) both models combined binomial probabilities in their estimation with either negative binomial or positive regression model. They are otherwise referred to as 2-part models (Data generating process).

Considering the number of maternal death mentioned by the number of antennal visits, a pregnant woman might have had zero antennal clinical visits during the period under consideration maybe as a result of the two data generating processes. Therefore, the study consider the following Zero-inflated models

(ii) Zero-Inflated Poisson Model

According to Yusuf and Ughali, (2015), the zero-inflated Poisson Regression model was introduced by Lambert in 1992 and this model according to Hur, Hedeker, Henderson, Khuri and Daley (2002) allows for covariates for both the binary and Poisson parts of the model. This is commonly used to model count data with excess zeros. The assumption of this model is that with probability if the only possible observation is zero and with probability $(1-\mu)$ a Poisson (λ) distribution is observed. The other count (data) generating process, for a pregnant woman which could visit Anatenal facility, has a Poisson with parameter $\lambda = \mu$ and the probability mass function expressed as thus

$$
g(y_i) = \frac{\exp(-\mu_i)\mu_i^{y_i}}{y_i!}
$$
 (3.41)

This follows that the numbers of visits y has a Poisson distribution with a conditioning means that depends on an individual's x_i such that this follows that the number of visits y has a Poisson distribution with a condition y mean that depends on an individual's x_i such that;

$$
E_n(y_i/x_i) = \mu_i = \lambda_i \quad \text{Where } \log(\lambda_i) = x_i\beta \ll \gg \lambda_i = \exp(x_i^1 \cdot \beta)
$$

With the probability mass function of count process as refine in equation $(*)$, the zero-inflation model is defined as thus;

$$
\rho(y_i / x_i, z_i) = \left\{ F_i + (1 - F_i) \exp(-\mu_i), y_i = 0 \quad (1 - F_i) \frac{\exp(\mu_i) \mu_i^{y_i}}{y_i!}, y_i \ge 0 \right\}
$$
(3.42)

The conditional mean of y_i and conditional variance are respectively expressed as thus; $E(y_i / x_i, z_i) = \mu_i (1 - F_i), \text{Var}(y_i / x_i, z_i) = E(y_i / x_i, z_i) (1 + \mu_i F_i)$

(iii) Zero – Inflated Negative Binomial Model:

Green (1994) described zero-inflation negative Binomial model as an extended form of the negative binomial regression models for excess zero count data. This is revived from a Poisson distribution with parameter λ - μ and its corresponding mass function as it is shown above. Hence, by specifying the negative binomial distribution for the second count in the generation process as shown above, we have

$$
g(y_i) = \frac{\Gamma\left(\frac{1+\alpha y_i}{\alpha}\right)}{\Gamma\left(\frac{1}{\alpha}\right)y_i!} \left(\frac{1}{1+\alpha\mu}\right)^{\frac{1}{\alpha}} \left(\frac{\alpha\mu}{1+\alpha\mu_i}\right)^{y_i}
$$
(3.43)

The zero-inflation negative Binomial (ZINB) model is thus given as follows:

The zero-inflation negative Binomial (ZINB) model is thus given as follows:
\n
$$
\rho(y_i/x_i, z_i) = \left\{ F_i + (1 - F_i) \left(\frac{1}{1 - \alpha \mu} \right)^{\frac{1}{\alpha}}, y_i = 0 \left(1 - F_i \right) \frac{\Gamma \left(\frac{1 + \alpha y_i}{\alpha} \right)}{\Gamma \left(\frac{1}{\alpha} \right) y_i!} \left(\frac{1}{1 - \alpha \mu_i} \right)^{\frac{1}{\alpha}} \left(\frac{\alpha \mu_i}{1 - \alpha \mu_i} \right)^{y_i} y_i \ge 1 \right\}
$$
\n(3.44)

Where α (alpha) is the over-dispersion parameter since increasing α increases the conditional variance of y and both the conditional of yⁱ and conditional variance are shown respectively to be as thus:

 $E(y_i/x_i, z_i) = \mu_i (1 - F_I)$ and $V(y_i/x_i, z_i) = E(y_i/x_i, z_i) = [1 + \mu_i (F_I + \alpha)]$

Note: supposing $\alpha = 0$, it then means that the mean and variance are the same, and we have a Poisson model. According to Yusuf et al, (2018), the zero inflation Poisson and zero-inflation Negative Binomial models are used to correct over-dispersion that arises when the variance is greater than the conditional mean.

(iv) Truncated Regression Models

In the estimation as well fitting data to a trimated regression model, it the distribution of the error terms in the latent variable model is assured to be known. Then, the most common assumption of Herror distribution terms are said to be normally independently and identically distributed as such the latent variables $X_t \beta$ with it probability Y_t^o is included in the simples. Hence;

$$
P_r(Y_t^0 > \delta) = P(X_t \beta + \mu_t > \delta)
$$

=1-Pr(μ_t - X_t β) = 1 - Pf($\frac{\mu_t}{\sigma} < -X_t \frac{\beta}{\sigma}$) = 1- $\Phi\left(-X_t \frac{\beta}{\sigma}\right) = \Phi\left(-X_t \frac{\beta}{\sigma}\right)$ (3.45)

Where $Y_t^o \geq 0$ and y_t is observed, the density of y_t is proportional to the density of y_t is proportional to the density of Y_t^o . Otherwise, the density of $y_t = \delta$. The factor of proportional, considered to ensuring that the density of y_t integrates to unit it become the inverse of the probability that $y_t^0 > 0$.

Therefore, the density of y_t can be written

$$
\frac{\sigma^{-1}\phi\left(y_t - X_t \frac{\beta}{\sigma}\right)}{\phi\left(X_t \frac{\beta}{\sigma}\right)}
$$

This simply implies that the log likelihood function, which is the sums of the overall t of the log of the density of y_t condition ∞ n $yt \ge 0$ become

$$
\rho(Y,\beta,\sigma) = -\frac{n}{2}\log(2\pi) - n\log(\sigma) - \frac{1}{2\sigma^2}\sum_{t}^{n}(Y_t - X_t\beta) - \sum_{l=1}^{n}\log\Phi\left(X_t\frac{\beta}{\sigma}\right) \tag{3.46}
$$

3.4 Model Selection Test

Model selection shall be done using two criteria which include: the Akaike information criteria (AIC) and Bayesian information criteria (BIC). The Akaike information criteria (AIC) is one of the most suitable and commonly used fitness statistics test (Hube, 2014). According to Nwankwe et al, (2016), it measure of the relative quality of a statistical model for a given set of data. In a given set of statistical models used in estimating and fitting a data set, the most preferred model among the set of statistical model is the one with the minimum AIC value i.e. the model with the smallest AIC value is the best model. It does not only reward model goodness-of-fit but also levies penalty on an increasing function of the number of estimated parameters (Deebom and Didi, 2017). It is refine as shown below in the two formulae:

$$
AIC(n) = \frac{-2}{n}[L - K] \qquad \text{and} \quad AIC(1) = -2[L - K]
$$

Where K is the number of predictors including the intercept, while AI(1) is usually an output by statistical of software applications. L is the maximized value of the likelihood function for the model

Similarly, Bayesian information criteria (BIC) according to Hube (2014) have three for mutations and they include as it is define in (Schwart, 1978).

$$
BIC(R) = D - (df) \ln (n)
$$

$$
BIC(L) = -2L + Kin(n)
$$

$$
BIC(Q) = \frac{-2}{n} (L - Kin (K))
$$

Where df is the residual degree of freedom

BIC (L) is given as SC in SAS and BIC in other software . L represents the log likelihood

3.5 Source of Data

Data used in the study was sourced for and extracted from the Rivers state Hospital Management Board Record. The Variables comprises of weekly data extracted from 2013 - 2017, making it a total of 357 data points. The variables of interest for the study are classified into two: Direct and Indirect causes maternal mortality other conditions incorporated into the classifications were supervised deliveries, maternal mortality ratios, delivery type and outcome of delivery, duration of stay at the hospital, gravid, parity and age. Statistical package STATA version 14 was used in analyzing the data. The Direct causes of maternal mortality were said to include those resulting from obstetric complications as at the pregnancy state. For examples, Obstetric, hemorrhages or hypertensive disorders, anesthesia, caesarean sections. Indirect Causes of maternal Mortality were those deaths resulting from previously existing diseases or conditions developed within the pregnancy period or an aggravation from existing conditions. For example an existing cardiac or renal diseases, ecclampsias, Sepsis, obstructed labor, Malaria related sickness. Also, maternal mortality could occur by one or multiple causes as classified above, however, 1 is indicated in the data extracted; this means the cause of maternal death is one and can be either of the causes listed above. Similarly, where $n \geq 2$ means that there are two or multiple causes of maternal mortality and so on.

3.6 Model Estimation Procedure/ Techniques Summary Descriptive Statistics

This is done to ascertain the summary descriptive statistic involving the mean and variance.

Multi-Collinearity Test Statistics

According to Nwankwo *et al.,* (2016), multi-nonlinearity test statistics otherwise referred to as Collinearity is a statistical occurrence in which two or more predictor variables in multiple regression models are largely corrected. This simply means that one can be linearly predicted from the others with non-trivial degree. Conversely collinearity can be said to occur when two or more independent variable are highly correlated. Ayunanda et al (2013) observed that this needed as an initial assumption for parameter estimation and one informal way of revealing this is by the use of the variance inflation factors (VIF). This is said to occur if the variance inflation factor value is greater than all the variables can be included in the subsequent analysis modeling with Poisson regression, generalized Poisson regression etc.

The variance inflated factor (VIF) used in testing for the presence of multi-collinearity is define

as this : $VIF = \frac{1}{1 - R_i^2}$ 1 R^2_j *VIF* - $=\frac{1}{1-R^2}$, Where R_f^2 represent the co-efficient of determination of a regression

of the explanatory variable j on all the other explanations.

3.7 Parameter Estimation of the Counts Models Family

The count models (Poisson regression, negative, Binomial Zero-inflation and truncated regression models) Estimation of Parameters and done to be sure of the co-efficient of each of the model specified in the study.

4.1 Results

The data analyzed are presented under the following sub-headings: Summary Descriptive statistics of the variables used in the study, Bar Charts, Multi-collinearity test results, Estimation Results of Poisson Regression, Poisson Generalized Regression, Negative Binomial Regression, Truncated Regression, Zero-inflated Poisson Regression, Zero- inflated Negative Binomial Regression Model, Estimation Results for Model selection and Fitness, Model Diagnostic Tests.

Source: Researcher's Computation, 2018 using STATA Version 14

International Journal of Applied Science and Mathematical Theory ISSN 2489-009X Vol. 5 No. 1 2019 www.iiardpub.org

Source: Researcher's Computation, 2018 using STATA Version 14.

Source: Researcher's Computation, 2018 using STATA Version 14.

Table 4.4: Estimation Results for Zero-inflated Poisson Regression and Zero- inflated Negative Binomial Regression Model

Source: Researcher's computation, 2018 using STATA version 14.

Table 4.5: Estimation Results For Model Selection Fitness and It Extensions with Their Corresponding AIC, BIC and -2log-Likelihood (-2ll).

Source: Researcher's computation, 2018 using STATA version 14. Overall Best fitted Model: Zero –Inflated Negative Binomial Regression

6.2 Conclusion

The research was aimed at first, to examine the significance of the occurrence and incidence of maternal mortality in Rivers state and secondly to assess the factors that may likely contribute to maternal mortality in some selected Hospital in Rivers state. The Poisson regression, Poisson Generalized Regression, Negative Binomial regression, truncated Regression model, Zeroinflated Poisson and Zero-inflated Negative Binomial Regression for the occurrence of maternal mortality in Rivers state were considered. In conclusion, in analyzing as well as fitting maternal mortality data with Poisson regression model with unique sources of excess zeros such as in the case of some selected hospital facilities in Rivers state, the zero-inflated negative binomial regression should be considered appropriate in the estimation.

6.3 Recommendations

In the words of Jin (2008), opined that increases the risk and uncertainty of external transactions and predisposes a country to related risks. Considering the level of risk that accompanies pregnancy and the corresponding investment government has made in order to reduce maternal mortality which has not yield the much needed results. The study carefully advised and recommends to medical practitioner, statisticians, policy makers and the pregnant women the following:

- The Rivers state Government in collaboration with the state and federal Ministry of Health (MOH) should as a matter of urgency equip hospital and clinical facilities in the state with the necessary infrastructure and well-trained medical personnel (Gynaecologist) in order to deal with cases complications during pregnancy .
- The Rivers state and federal Ministry of Health (MOH) in Nigeria must provide clinics with three or more midwives in all the hospital and clinical facilities in the country.
- There should be sensitization and massive campaign on the need and importance of antennal visits for pregnant women.
- Government, stakeholders and policy makers should as a matter of urgency evaluate as well as do an appraisal all the current existing intervention maternal health programs since they seem not to have yielded the expected results within the past.
- \triangleright Since the study have revealed that direct cause of death such as parity, age and delivery type contributes significantly to maternal mortality at the selected health facilities in

Rivers State under the study. It is necessary for management of health facilities to put in more effort at implementing existing programs aimed at reducing the problems associated with maternal mortality.

References

- Abou Zahr, C. (2007). Safe motherhood: a brief history of the global movement 1947–2002. *British Medical Bulletin*, 67, 13-25.
- Alves, S.V. (2007).Maternal mortality in Pernambuco, Brazil: what has changed in ten years? *Reproductive Health Matters*, 15(30), 134-144.
- Arnold, R. (1968). *Reflections on a theory of modernization.* NY: Anchor Books.
- Ayunanda M, Yeni S, Eko. H, Rosi, S. & Purhadi (2013). The comparison of generalized Poisson regression & negative Binomial regression methods in overcoming over dispersion. *International Journal of Scientific & Technology research,* 2(8), 34-56.
- Bawa, R. (2008). Nanoparticle-based therapeutics in humans: a survey. *Nanotechnology Law & Business*, Summer, 43-68.
- Black, J. (2002). *Oxford dictionary of economics (2nd edi.).* New York: Oxford University Press Inc.
- Brestow, N.E. (1990). Testing of hypotheses in over dispersed Poisson regression and other quasi – likelihood models. *Journal of the American Statistical Association,* 85(1990), $565 - 571.$
- Cameron, A.C & Trivedi, P. K (1998). *Regression analysis of count data*. Cambridge: Cambridge University press.
- Chike, H. N & Godwin, I.(2016), Statistical model of road traffic crashes data in Anambra State, Nigeria: a Poisson regression approach. *European Journal of Statistics and Probability,* 4(1), 41-60.
- Christian, P., Katz, J., Wu, L., Kimbrough-Pradhan, E., Khatry, S.K., LeClerq, S.C. (2008). Risk factors for pregnancy-related mortality: a prospective study in rural Nepal. *Public Health,* 122(2), 161-72.
- Cook, J.D. (2009). Notes on the negative binomial distribution. (accessed on Feb. 15th, 2018). www.johndcook.com/negative_binomial.pdf.
- Downie, R., (2012). *The state of public health in South Sudan*, S.l: Centre for Strategic and International Studies, Global Health Policy Centre.
- Famoye, F. (1993). Restricted generalized Poisson regression model. *Communications in Statistics – Theory and Methods,* 22, 1335-1354.
- Ferraro, K. (1996). Women's fear of victimization: shadow of sexual assault? *Social Forces*, 75, 667–90.
- Garenne, M., Mbaye, K., Bah, M.D., & Correa, P. (2003). Risk factors for maternal mortality: a case-control study in Dakar Hospitals (Senegal). *African Journal of Reproductive Health,* 1(1), 14-24.
- Graham, W.J., Fitzmaurice, A.E., Bell, J.S., & Cairns, J.A. (2004). The familial technique for linking maternal death with poverty. *Lancet,* 363(9402), 23-47.
- Greene, W (2008). Functional forms for the negative Binomial model for count data. Foundations and Trends in Econometrics. Working Paper, Department of Economics, Stern School of Business, New York University. 585-590.
- Greenwood, M., & Yule, G.U. (1920). An inquiry into the nature of frequency distributions of multiple happenings, with particular reference to the occurrence of multiple attacks of disease or repeated accidents. *Journal of the Royal Statistical Society*, 83, 255-279.
- Gupta, P., Gupta, R., & Tripathi, R. (1996). Analysis of zero-adjusted count data. *Computational Statistics & Data Analysis, 23,*207–218. [https://doi.org/10.1016/S0167-](https://doi.org/10.1016/S0167-9473(96)00032-1) [9473\(96\)00032-1](https://doi.org/10.1016/S0167-9473(96)00032-1)

- Hilbe, J. M. (2011). Negative binomial regression (2nd ed.). Cambridge, UK; New York: Cambridge University Press.
- Hinde, J., & Demetrio, C. (1998). Over dispersion: models and estimation. *Computational Statistics & Data Analysis, 27,*151-170.
- Hu, M., Pavlicova, M., & Nunes, E. (2011). Zero-inflated and hurdle models of count data with extra zeros: Examples from an HIV-risk reduction intervention trial. *The American Journal of Drug and Alcohol Abuse*, 37(5), 367–375.
- Hur, K., Hedeker, D., Henderson, W., Khuri, S., & Daley, J. (2002). Modeling clustered count data with excess zeros in health care outcomes research. *Health Services & Outcomes Research Methodology, 3*, 5–20.
- Hur, K., Hedeker, D., Henderson, W., Khuri, S., & Daley, J. (2002). Modeling clustered count data with excess zeros in health care outcomes research. *Health Services & Outcomes Research Methodology, 3*, 5–20.
- ICDIO (2016). International statistical classification of disease on health problems, $10th$ version. Annual Meeting.
- Illah, E., Mbaruku, G., Masanja, H. & Kahn, K. (2013). Causes and risk factors for maternal mortality in rural Tanzania - case of Rufiji Health and Demographic Surveillance Site (HDSS). *African Journal of Reproductive Health September,* 17(3), 119-130.
- Jahromi, B.N. & Husseini, Z. (2008). Pregnancy outcome at maternal age 40 and older. *Taiwanese Journal of Obstetrics & Gynecology*, 47(3), 318-321.
- Kerlinger, F.N (1973). *Foundation of behavioral research*. New York: Holt, Rinehart and Winston
- Lambert, D. (1992). Zero-inflated Poisson regression with an application to defects in manufacturing. *Technometrics, 34*(1), 1-14.
- Lord, D., & Park, P.Y.J. (2008). Investigating the effects of the fixed and varying dispersion parameters of Poisson-Gamma models on empirical Bayes estimates. *Accident Analysis & Prevention*, 40(4), 1441-1457.
- Lord, D., Washington, S.P., & Ivan, J.N. (2005). Poisson, Poisson-Gamma and zero inflated regression models of motor vehicle crashes: balancing statistical fit and theory. *Accident Analysis & Prevention,* 37(1), 35-46.
- Mojekwu, J.N. (2005). Maternal mortality: natural risk to women. *Ghana Journal of Development Studies*, 2(1), 129-141.
- National Population Commission (NPC) [Nigeria] and ICT International (2014). *Nigeria Demographic and Health Survey 2013*. Abuja, Nigeria and Rockville, Maryland, USA: NPC and ICT International
- Nnamdi, M. & Ibekwe, U. (2012). Maternal mortality in Nigeria: examination of intervention methods. *International Journal of Humanities and Social Science*, 2(20), 134-149.
- Nsirim-Worlu, H.G. (2011). Gender stratification: a study of discrimination and oppression in selected communities in Nigeria. *Academic Research International*, 1(2), 238-245.
- Nwankwo, C. H & Nwaigwe ,G. I (2016), Statistical model of road traffic crashes data in Anambra State, Nigeria: a Poisson regression approach. European Journal of Statistics and Probability, $1(4)$, $41-60$
- Nworgu, B.G (2006). *Educational research basic issues and methodology*. Nssuka: University Trust Publishers.
- Ogbuokiri, C., & Egeonu, R. (2013). Maternal mortality at Nnamdi Azikiwe teaching hospital, southeast nigeria: a 10-year review (2003 - 2012). *International Journal of Women's Health, 5, 431-436.*
- Olatayo, T. I. & Adeboye, N.O. (2013). Predicting population growth through births and death rate in Nigeria. *Mathematical theory and Modelling*, 3(1), 96-101.
- Omoruyi, G. (2008). Causes of maternal mortality in Nigeria, Sunday observer, Benin.

- Onwumere, J.U.J. (2005). *Business and economic research methods*. Lagos: Don-Vinton Limited.
- Oyindamola, B., Yusuf, R., Afolabi, F. & Ayoola, S. A. (2018), Modelling excess zeros in count data with application to antenatal care utilization. *International Journal of Statistics and Probability, 7(3), 62-81*
- Ramos, S., Karolinski, A., Romero, M., & Mercer, R. (2007). A comprehensive assessment of maternal deaths in Argentina: translating multicentre collaborative research into action. *Bulletin of the World Health Organization,* 85(8), 615-22.
- Salifu, A.(2014), Analysis of maternal mortality in Wa District. *International Journal of Humanities and Social Science*, 6(3), 34-49.
- Sarpong, S. (2012). Analysis of maternal mortality with time: A case study of Okomfo Anokye Teaching Hospital in Kumasi. Master's thesis, Mphil Thesis, Department of Mathematics, Kwame Nkrumah University of Science and Technology
- Shiffman, J. & Okonofua, F. (2007). The state of political priority for safe motherhood in Nigeria. *Br. J. Obstet. Gynecol*, 114, 127-133.
- Sunday Punch (2011). Abiye safe delivery, December 25, 2011, 17(19670), 18-19.
- Tawiah, K (2015). Modeling Maternal Mortality In Ghana Using Hierarchical Models (2010- 2013),
- Ujah, I.A.O., Aisien, O.A., Mutihir, J.T., Vanderagt, D.J., Glew, R.H., & Uguru, V.E. (2005). Factors contributing to maternal mortality in North-Central Nigeria: a seventeen-year review. *African Journal Reproduction Health,* 9(3), 27-40.
- Wang, W.R. & Famoye, F. (1997). Modeling household fertility decisions with generalized Poisson regression. *Journal of Population Economics*,10(3), 273-283
- WHO (2015). Fac t sheets on sustainable development goals : health targets. Maternal health. Retrieved from http://www.euro.who.int/ data/assets/pdf file/0006/354921/3.1-[SDG-Fact-sheet-Maternal-Health.pdf?ua=1](http://www.euro.who.int/__data/assets/pdf_file/0006/354921/3.1-SDG-Fact-sheet-Maternal-Health.pdf?ua=1)
- WHO, (2016). Countries: South Sudan. [Online] Availableat:http://www.who.int/ countries/ssd/en/ [Accessed 8 January 2017].
- Wimberley, D.W. & Bello, R. (1992). Effects of foreign investment, exports, and economic growth on third world food consumption. *Social Forces,*70, 895-921.
- Wodi, B.E. (2005). HIV/AIDS knowledge, attitudes, and opinions among adolescents in the River States of Nigeria. *The International Electronic Journal of Health Education*, 8, 86-94.
- World Health Organization (2008), Factsheet, maternal mortality, Department of Making Pregnancy Safer.
- World Health Organization (2013). WHO global database on maternal health indicators, 2013 update. Geneva, World Health Organization. Retrieved from http://www.who.int/gho.
- Yusuf, O. B., & Ughali, L. (2018). On the performance of the Poisson, negative binomial and generalized Poisson regression models in the prediction of antenatal care visits in Nigeria. *American Journal of Mathematics and Statistics, 5*(3), 128-136.